Filters and smoothers

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Why to remove noise from signal?

- Control
  - signal noise in the control loop means vibrations in the control action, i.e. on the actuators, with higher power consumption and system degradation
  - filtering is causal
  - trade-off between noise reduction and phase shift (“delay”) introduced by the filter
  - model-based prediction can reduce filter “delay”

- Sensor characterisation
  - an estimate of the signal noise is useful for realistic simulation of the sensors and for tuning filter parameters
  - smoothing is not causal

Sensor characterisation

- Hp. neglecting sensor time constant
  - see later, in Motion Estimation, the example of vision and clinometers
- Signal processing steps
  - removing outliers
  - determining the bandwidth of the system
  - smoothing the signal
  - computing the *noise*

Tools: outlier removal

- **Recursive symmetric median filter with MAD-based threshold**
- median: the value separating the higher half of a data sample from the lower half
- MAD: Median Absolute Deviation
- Given the signal x_k and its estimate $\hat{x}_k$, consider the window of 2N samples centered at time k
- $$W_k = \{\hat{x}_{k-N+1}, \ldots, x_k, \ldots, x_{k+N-1}\}$$

$$\bar{x}_k = \text{median}(W_k) \quad S_k = \text{MAD}(W_k) \quad d_k = |x_k - \bar{x}_k|$$

$$T_k = \min(\max(c * S_k, T_{min}), T_{max})$$

$$\text{if} \quad d_k > T_k \quad \text{then} \quad \hat{x}_k = \bar{x}_k$$

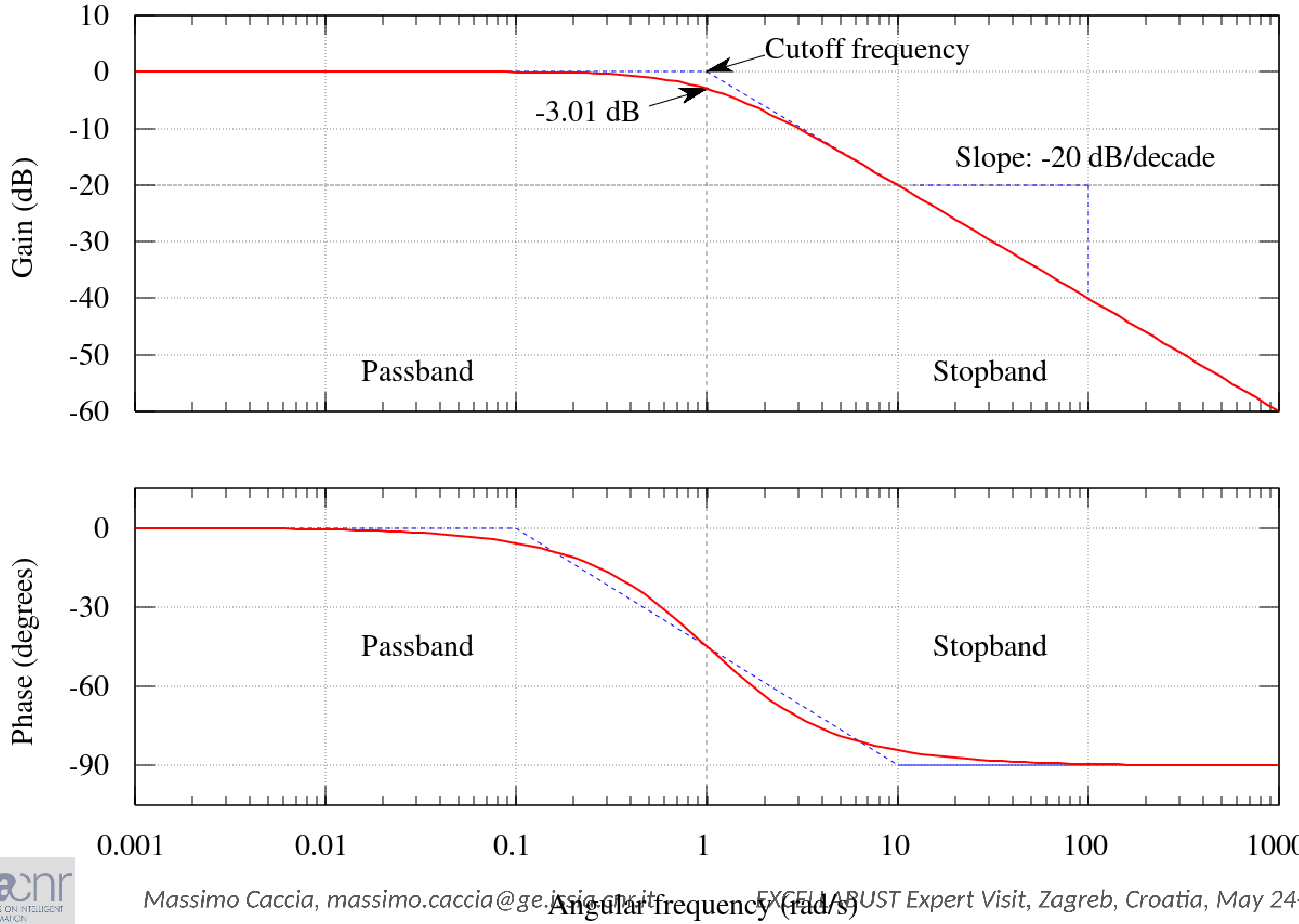
outlier

constrained adaptive threshold

Tools: Low Pass Filtering

- **Butterworth filter: designed to have as flat a frequency response as possible in the pass-band**
- In order to obtain a zero-phase filtering, the matlab function ***filtfilt*** can be used
    - After filtering in the forward direction, the filtered sequence is then reversed and run back through the filter; Y is the time reverse of the output of the second filtering operation. The result has precisely zero phase distortion, and magnitude modified by the square of the filter's magnitude response. Startup and ending transients are minimized by matching initial conditions.

Bode plot of a first order Butterworth LPF



Kalman filter

state estimator of a linear dynamic system from a series of noisy measurements

Assumptions

state transition $\underline{x}_k = F_k \underline{x}_{k-1} + B_k u_k + \boxed{w_k}$

system noise: Gaussian with covariance Q_k

observation $\underline{z}_k = H_k \underline{x}_k + \boxed{v_k}$

measurement noise: Gaussian with covariance R_k

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Kalman filter

Basic filter structure

Prediction

$$\underline{\hat{x}}_{k/k-1} = F_k \underline{\hat{x}}_{k-1/k-1} + B_{k-1} u_{k-1}$$

$$P_{k/k-1} = F_k P_{k-1/k-1} F_k^T + Q_k$$

system noise increases the uncertainty of the prediction

Innovation (measurement residual)

difference between the observation and its estimate

$$\underline{\epsilon}_k = \underline{z}_k - H_k \underline{\hat{x}}_{k/k-1}$$

innovation covariance and filter gain computation

$$S_k = H_k P_{k/k-1} H_k^T + R_k \qquad G_k = P_{k/k-1} H_k^T S_k^{-1}$$

Updating of the estimate

$$\underline{\hat{x}}_{k/k} = \underline{\hat{x}}_{k/k-1} + G_k \underline{\epsilon}_k$$

$$P_{k/k} = (I - G_k H_k) P_{k/k-1}$$

Extended Kalman filter

the state transition and observation models do not need to be linear functions of the state but may instead be differentiable functions

$$\underline{x}_k = f(\underline{x}_{k-1}, u_k) + w_k$$
$$\underline{z}_k = h(\underline{x}_k) + v_k$$

non-linear version of the Kalman filter which linearises about the current mean and covariance

$$F_k = \frac{\partial f}{\partial \underline{x}}(\underline{\hat{x}}_{k-1/k-1}, \underline{u}_k) \qquad H_k = \frac{\partial h}{\partial \underline{x}}(\underline{\hat{x}}_{k/k-1})$$

the estimated covariance matrix tends to underestimate the true covariance matrix and therefore risks becoming inconsistent in the statistical sense without the addition of "stabilising noise"